Homework #5

There are many different tests to check the randomness of a table of digits. In this exercise we will discover two of them and use them to study whether the digits of π can be considered random or not.

Part A – The maximal blocks test

For this test, the digits are grouped in three-digits blocks. A block of three consecutive digits in the table is said to be maximal when the middle digit is strictly greater then the two other digits. For example, blocks $142, 486, 053$ are maximal whereas blocks 426 and 663 aren't.

1. How many different three-digits blocks are there ?
2. (a) If a block is maximal and its tens digit is 6, what are the possible values for the units digit and the hundreds digit.
   (b) Deduce the number of maximal blocks with the tens digit equal to 6.
3. Use the same method to fill out the table below .

Tens digit	0	1	2	3	4	5	6	7	8	9
# of maximal blocks							36			

4. Deduce the probability of occurence of a maximal block in a random table of digits.

Part B – The poker test

One of the best tests to check the randomness of a table of digits is the poker test.
Digits are grouped in five-digits blocks, divided into 7 categories, or hands :

- busts : *abcde* (exemple : 03251) ;
- pairs : *aabcd* (exemple : 80919) ;
- two pairs : *aabbc* (exemple : 08608) ;
- threes : *aaabc* (exemple : 12151) ;
- full house : *aaabb* ;
- fours : *aaaab* ;
- fives : *aaaaa*.

The order of the digits inside the block is not considered. For example, the blocks 12345 and 13542 are the same. The table below gives the probability of each type of hand.

Hand	*abcde*	*aabcd*	*aabbc*	*aaabc*	*aaabb*	*aaaab*	*aaaaa*
Probability	0.3024	0.5040	0.1080	0.0720	0.0090	0.0045	0.0001

1. How many different five-digits block are there ?
2. In a no-pair hand, how many possible values are there for a, for b, for c, for d and for e ? Deduce the number of possible hands of this kind and check the probability given in the table above.
3. Explain the probability given for the fives.

Part C – The digits of π

In this part, we will use the two tests to study the randomness of the digits of π appearing after the decimal point. Here are the first 1005 digits of π. .

3,141 592 653 589 793 238 462 643 383 279 502 884 197 169 399 375 105 820 974 944 (1)
592 307 816 406 286 208 998 628 034 825 342 117 067 982 148 086 513 282 306 647 (2)
093 844 609 550 582 231 725 359 408 128 481 117 450 284 102 701 938 521 105 559 (3)
644 622 948 954 930 381 964 428 810 975 665 933 446 128 475 648 233 786 783 165 (4)
271 201 909 145 648 566 923 460 348 610 454 326 648 213 393 607 260 249 141 273 (5)
724 587 006 606 315 588 174 881 520 920 962 829 254 091 715 364 367 892 590 360 (6)
011 330 530 548 820 466 521 384 146 951 941 511 609 433 057 270 365 759 591 953 (7)
092 186 117 381 932 611 793 105 118 548 074 462 379 962 749 567 351 885 752 724 (8)
891 227 938 183 011 949 129 833 673 362 440 656 643 086 021 394 946 395 224 737 (9)
190 702 179 860 943 702 770 539 217 176 293 176 752 384 674 818 467 669 405 132 (10)
000 568 127 145 263 560 827 785 771 342 757 789 609 173 637 178 721 468 440 901 (11)
224 953 430 146 549 585 371 050 792 279 689 258 923 542 019 956 112 129 021 960 (12)
864 034 418 159 813 629 774 771 309 960 518 707 211 349 999 998 372 978 049 951 (13)
059 731 732 816 096 318 595 024 459 455 346 908 302 642 522 308 253 344 685 035 (14)
261 931 188 171 010 003 137 838 752 886 587 533 208 381 420 617 177 669 147 303 (15)
598 253 490 428 755 468 731 159 562 863 882 353 787 593 751 957 781 857 780 532 (16)
171 226 806 613 001 927 876 611 195 909 216 420 198 938 095 (17)

1. Apply the maximal blocks test on this table and give clearly the result.
2. Apply the poker test on this table and give clearly the results.
3. Do you think that this table of digits can be considered as random ?
4. Why did we choose to study 1005 digits, and not just 1000 ?