Homework #6

Let ABC be an equilateral triangle with side 3 cm. For the figure, A will be placed on top of the triangle and on top of the paper sheet, with B on the right-hand side and C on the left-hand side.

1. Place the point D such that $\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{AC}$.
2. Place the point E such that $\overrightarrow{AE} = 3\overrightarrow{AC}$.
3. Place the point F such thate $\overrightarrow{AF} = 2\overrightarrow{AB} + \overrightarrow{AC}$.
4. Place the point M such that $\overrightarrow{MA} = -2\overrightarrow{AB}$.
5. Let N be the point such that $\overrightarrow{NA} = 3\overrightarrow{BA}$.
   (a) Decompose $\overrightarrow{AN}$ using only $\overrightarrow{AB}$ and $\overrightarrow{AC}$
   (b) Place the point N.
6. Let P be the point such that $\overrightarrow{PA} + \overrightarrow{PB} = 4\overrightarrow{CA} + \overrightarrow{BA}$.
   (a) Decompose $\overrightarrow{AP}$ using only $\overrightarrow{AB}$ and $\overrightarrow{AC}$
   (b) Place the point P.
7. Let Q be the point such that $\overrightarrow{QA} + \overrightarrow{QB} + \overrightarrow{QC} = 8\overrightarrow{CA} + 2\overrightarrow{BA}$.
   (a) Decompose $\overrightarrow{AQ}$ using only de $\overrightarrow{AB}$ and $\overrightarrow{AC}$
   (b) Place the point Q.
8. Using the previous questions and Chasles' relation, decompose $\overrightarrow{DB}$, $\overrightarrow{DP}$ and $\overrightarrow{DQ}$ using $\overrightarrow{AB}$ and $\overrightarrow{AC}$. What can you deduce about the points B, D, P,Q ?
9. (a) Place the point I, midpoint of the segment $[FP]$.
   (b) Find out a relation between the vectors $\overrightarrow{FI}$ and $\overrightarrow{FP}$.
   (c) Write $\overrightarrow{FP}$ and $\overrightarrow{FI}$ using $\overrightarrow{AB}$ and $\overrightarrow{AC}$.
   (d) Use $\overrightarrow{AF}$ and $\overrightarrow{FI}$ to decompose $\overrightarrow{AI}$ with $\overrightarrow{AB}$ and $\overrightarrow{AC}$.
   (e) Prove that $\overrightarrow{AD} = \frac{2}{3}\overrightarrow{AI}$. What can you deduce about the point D in the triangle AEN ?
   (f) Deduce from the previous questions the intersection point of (ND) and (AE) and the intersection point of the lines (ED) and (AN).