Homework # 10

The aim of this homework is to study the famous Euler line on an example.
Consider an orthonormal coordinate graph $(O; \vec{i}, \vec{j})$ with the points $A(-3, 1)$, $B(5, 1)$ and $C(-2, 8)$. We call A' the midpoint of $[BC]$ and B' the midpoint of $[AC]$.

1. For the following questions, you will need to find information on the internet or in maths books.
   (a) Who was Euler ? When did he live ?
   (b) What is the Euler line of a triangle ?
   (c) Draw the triangle ABC and its Euler line.
2. (a) Compute the coordinates of A' and B'.
   (b) Find out equations of the medians through A and B in triangle ABC.
   (c) Deduce the coordinates of G, the centroid of ABC.
3. Consider the point $R(1, 4)$.
   (a) Compute the lengths RA, RB and RC.
   (b) What is the point R in the triangle ABC ?

We now admit that two lines Δ and Δ' with respective slope-intercept equations $y = mx + p$ and $y = m'x + p'$ are perpendicular if and only if $m \times m' = -1$.

4. (a) What is the slope of any line perpendicular to (BC).
   (b) Deduce the slope-intercept equation of the altitude through A in ABC.
   (c) Find out the simplest equation of the altitude through C in triangle ABC.
   (d) Deduce the coordinates of H, the orthocenter of the triangle ABC.
5. Use vectors to check that the points G, R and H are collinear.
6. The Euler line passes through a fourth important point in the triangle. Find the name and definition of this point.