Épreuve de section européenne

1 General knowledge

How can you prove that two triangles are congruent ?

2 Document

"You can never catch up."

> "In a race, the quickest runner can never overtake the slowest, since the pursuer must first reach the point whence the pursued started, so that the slower must always hold a lead." (Aristotle Physics VI :9, 239b15)

In the paradox of Achilles and the tortoise, we imagine the Greek hero Achilles in a footrace with the plodding reptile. Because he is so fast a runner, Achilles graciously allows the tortoise a head start of a hundred feet. If we suppose that each racer starts running at some constant speed (one very fast and one very slow), then after some finite time, Achilles will have run a hundred feet, bringing him to the tortoise's starting point ; during this time, the tortoise has "run" a (much shorter) distance, say one foot. It will then take Achilles some further period of time to run that distance, during which the tortoise will advance farther ; and then another period of time to reach this third point, while the tortoise moves ahead. Thus, whenever Achilles reaches somewhere the tortoise has been, he still has farther to go. Therefore, Zeno says, swift Achilles can never overtake the tortoise.

From *Wikipedia*, the free encyclopedia.

3 Questions

1. Draw a quick sketch of this situation.
2. What do you think of this phenomenon ? Is it true ?
3. What mathematical tools can be used to modelize this situation ?
4. Suppose that the tortoise runs at a constant speed of v meters per second (ms^{-1}) and gets a head start of distance d meters, and that Achilles runs at constant speed xv ms^{-1} with $x > 1$.
   a. How long will it take for Achilles to reach the point where the tortoise started ?
   b. What distance will the tortoise have travelled at this time ?
   c. How long will it take for Achilles to cover this additionnal distance ?
   d. We admit that the time taken for Achilles to catch up with the tortoise is the limit
   $$\lim_{n\to+\infty} \frac{d}{v} \sum_{k=0}^{n} \left(\frac{1}{x}\right)^k.$$
   Is this limit a real number ? Conclude.