Épreuve de section européenne

1 General knowledge

Give the definitions and a few properties of the medians and angle bisectors of a triangle.

2 Document

Equivalence relations

A relation [between two elements of a set] is usually described by a statement which involves an arbitrary pair of elements of the set. For example the statement "x is the mother of y" describes a relation on the set P of living people ; for certain pairs $(x; y)$ of people the statement is true, and for other pairs it is false. Similarly "$x > y$" describes a relation on the set $\mathbf{R}$ of all real numbers. As a general notation we shall often use $\simeq$ for a relation on a set A.
$\simeq$ is a relation on the set A if, for each pair $(x; y)$ of elements of A, the statement "$x \simeq y$" has a meaning, i.e. it is either true or false for that particular pair.

Definition 1
$\simeq$ is a reflexive relation if it satisfies the condition E1 :

if x is any element of A, then $x \simeq x$.

Definition 2
$\simeq$ is a symmetric relation if it satisfies E2 :

if x, y are any elements of A such that $x \simeq y$, then also $y \simeq x$.

Definition 3
$\simeq$ is a transitive relation if it satisfies the condition E3 :

if x, y, z are any elements of A such that $x \simeq y$ and $y \simeq z$, then also $x \simeq z$.

Definition 4
$\simeq$ is an equivalence relation if it satisfies all three conditions E1, E2 and E3.

From *Sets and groups*, by J.A. Green.

3 Questions

1. Show that the relation of equality, denoted $=$, on the set of real numbers $\mathbf{R}$, is an equivalence relation.
2. Can we define a relation $=$ on the set P of all people, by taking $x \simeq y$ to mean "x and y have the same age" ? If so, check that it is an equivalence relation.
3. If we define $\simeq$ to be the relation "is the mother of" on the set P, is $\simeq$ a reflexive relation ? Is it symmetric ? Is it transitive ?
4. What about the relations "$<$" and "$\leqslant$" ?