Épreuve de section européenne

1 General knowledge

How can you prove the existence of a maximum of a function f ?

2 Document

Pascal's triangle

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

This triangular array is called Pascal's triangle. Each row gives the binomial coefficients. That is, the row 1 2 1 is made of the coefficients of $(a+b)^2$. The next row, 1 3 3 1, is made of the coefficients of $(a+b)^3$, and so on.
To construct the triangle, write 1, and below it write 1 1. Begin and end each successive row with 1. To construct the intervening numbers, add the two numbers immediately above.
Thus to construct the third row, begin it with 1, then add the two numbers immediately above : $1+1$. Write 2. Finish the row with 1.
To construct the next row, begin it with 1, and add the two numbers immediately above : $1+2$. Write 3. Again, add the two numbers immediately above : $2+1=3$. Finish the row with 1.

The $k-$th binomial coefficient of line n is noted $\binom{n}{k}$. Hence, in the expansion of $(a+b)^n$, each term has the form

$$\binom{n}{k} a^{n-k} b^k$$

where k successively takes on the values 0, 1, 2,..., n.

Adapted from *The Mathpage.com*.

3 Questions

1. Use the method explained in the text to build the eighth line of the triangle.
2. Expand $(x-1)^4$ using Pascal's triangle.
3. Find out the coefficient of a^2b^7 in the expansion of $(a+b)^9$. Which other term has the same coefficient ?
4. What do you get if you add the numbers in a row ? Explain why.
5. Use factorials to write the general term in the expansion of $(a+b)^n$.
6. Write the first four terms of $(x+h)^n$ without using factorials.