Épreuve de section européenne

1 General knowledge

What is the definition of the derivative of a function at a point ? What is the relationship between this number and the tangent to the curve at a point ?

2 Document

The mathematical constant $\pi \approx 3.14159$ is the ratio of a circle's circumference to its diameter in Euclidean geometry, and has many uses in mathematics, physics, and engineering. The Greek letter π is used to denote this number because it is the first letter of Greek $\pi\epsilon\rho\iota\phi\epsilon\rho\iota\alpha$ (periphery) and $\pi\epsilon\rho\iota\mu\epsilon\tau\sigma\nu$ (perimeter).

The number π is irrational ; that is, it cannot be written as the ratio of two integers, as was proven in 1761 by Johann Heinrich Lambert.

The number π is also transcendental, as was proven by Ferdinand von Lindemann in 1882. This means that there is no polynomial with rational coefficients of which π is a root. An important consequence of the transcendence of π is the fact that it is not constructible. Because the coordinates of all points that can be constructed with compass and straightedge are constructible numbers, it is impossible to square the circle, that is, it is impossible to construct, using compass and straightedge alone, a square whose area is equal to the area of a given circle.

Adapted from *Wikipedia*, the free Encyclopedia.

3 Questions

1. What is the precise definition of the number π ?
2. Why is the Greek letter π used to name this number ?
3. The fraction $\frac{22}{7}$ has often been used for π. Is it an exact value ?
4. Do you know another irrational number ? Can you prove that it is so ?
5. Is the real number $\sqrt{2}$ transcendental ? If not so, how can you construct it using compass and straightedge alone ?