Épreuve de section européenne

Probability and Ambiguity

Charles Sanders Pierce once observed that in no other branch of mathematics is it so easy to blunder as in probability theory. History bears this out. Leibniz thought it just as easy to throw 12 with a pair of dice as to throw 11. Jean le Rond d'Alembert, the great 18th-century French mathematician, could not see that the results of tossing a coin three times are the same as tossing three coins at once, and he believed (as many amateur gamblers persist in believing) that after a long run of heads, a tail is more likely.

From *The Colossal Book of Mathematics*, by Martin Gardner.

Questions

1. What's the main point made in this text ? What do you think about it ?
2. Give the probability of throwing a 12 with a pair of dice and the probability of throwing a 11. Was Leibniz right ?
3. Do you agree with the writer's assumption that "the results of tossing a coin three times are the same as tossing three coins at once" ?
4. A coin is tossed three times in a row. We note H_1 the event "the first throw produces heads", T_1 the event "the first throw produces tail". The events R_2, T_2, R_3 and T_3 are defined in the same way for the second and third throws.
   a. Draw a probability tree to represent this situation.
   b. Give the probability of T_1.
   c. Compute the probabilities of the events $T_1 \cap T_2$ and $R_1 \cap T_2$ and deduce the probability of T_2.
   d. Compute in the same way the probability of T_3.
   e. Was d'Alembert right about a tail being more likely after a long run of heads ?