Baccalauréat, série S Session de juin 2008

Épreuve de section européenne

The mathematics of continuity

Topology [...] is the mathematics of continuity. Continuity is the study of smooth, gradual changes, the science of the unbroken. Discontinuities are sudden, dramatic : places where a tiny change in cause produces an enormous change in effect. A potter, moulding a lump of clay in his hands, is deforming it in a continuous fashion ; but when he breaks a lump of clay off, the deformation becomes discontinuous. Continuity is one of the most fundamental mathematical properties of them all, so natural a concept that its basic role only became clear a hundred years or so ago, so powerful a concept that it is transforming mathematics and physics, so elusive[1] a concept that even the simplest questions took decades to answer.

From *Does God play dice ?* by Ian Stewart

Questions

1. What is the author's intuitive definition of continuity ?
2. What is, for the author, the conceptual difference between a continuous and discontinuous process ?
3. Explain the metaphors the author uses to illustrate the concept of continuity.
4. Give an example of a continuous function over $\mathbf{R}$ and an example of a discontinuous function defined over $\mathbf{R}$.
5. Let f be a function defined over $\mathbf{R}$ and a a real number. Give the definition of the fact that f is continuous at a.
6. Continuity arises in an important theorem that you've seen this year. State out this theorem and give an example of how it can be used.

[1] difficult to understand