Baccalauréat, séries ES et S — Session de juin 2008

Épreuve de section européenne

How to compute areas and slopes ?

A graph showing how the body's speed varies with time takes the form of a curve. By geometric arguments it can be shown that the total distance travelled is equal to the *area* under the curve. Similarly the velocity is the slope to the *tangent* to another graph, this time plotting distance against time. But how do we find these areas and tangents ? Newton, and independently Gottfried Leibniz, solved these problems by dividing time into tinier and tinier intervals. The area under a curve then becomes the sum of the areas of a large number of narrow vertical strips. They showed that the error made by such an approximation becomes very tiny as the time interval becomes smaller and smaller, and argued that "in the limit" the error can be made to vanish altogether. In the same way, the slope of a tangent can be calculated by considering two nearby time values and letting the difference between them become arbitrarily small.

From *Does God play dice ?* by Ian Stewart

Questions

1. Illustrate graphically the first two sentences of the text.
2. Explain with a drawing the "rectangle" method described in this textto find the area under a curve.
3. What is the mathematical notation for the area under the graph of a positive function f, delimited by the x-axis, the y-axis ad the line of equation $x = 2$?
4. Explain with a drawing the method described in this text to compute the slope of a tangent.
5. Let f be a function defined over $\mathbf{R}$. What is the name of the function defined by taking for each $x \in \mathbf{R}$ the slope of the tangent to the graph of f at x (if possible) ?
6. Consider the function g defined over $\mathbf{R}$ by $g(x) = -e^{3x-6} + x^2$. Compute $g'(x)$. Compute the slope of the tangent at $x = 2$. Give an equation of this tangent.