Épreuve de section européenne

Division by 0

$\frac{0}{0}$ is indeterminate. For the quotient of two numbers is the number which multiplied by the divisor will give the dividend. But any number whatever multiplied by zero gives zero, and the quotient is indeterminate; that is, any number whatever may be considered as the quotient, a result which is of no value.

$\frac{a}{0}$ has no meaning, a being different from zero, for there exists no number such that if it be multiplied by zero, the product will equal a.

Therefore division by zero is not an admissible operation.

Care should be taken not to divide by zero inadvertently. The following fallacy[1] is an illustration.

Assume that $a = b$.
Then evidently $ab = a^2$.
Subtracting b^2, $ab - b^2 = a^2 - b^2$.
Factoring, $b(a - b) = (a + b)(a - b)$.
Dividing by $a - b$, $b = a + b$.
But $a = b$, therefore $b = 2b$, or, $1 = 2$.

From *Elements of the Differential and Integral Calculus* by William Anthony Granville

Questions

1. Explain how the author shows that division by 0 is an illegal operation.
2. Why does the author study two different cases of division by 0 ?
3. What can you conclude from the last equality in the illustration ?
4. What is the mistake in the "illustration". How does it illustrate the point made by the author ?

[1] A fallacy is an argument, or apparent argument, which professes to be decisive of the matter at issue, while in reality it is not.