Épreuve de section européenne

Legend of the Ambalappuzha Paal Payasam

According to the legend, Lord Krishna once appeared in the form of a sage in the court of the king who ruled the region and challenged him for a game of chess (or chaturanga). The king being a chess enthusiast himself gladly accepted the invitation. The prize had to be decided before the game and the king asked the sage to choose his prize in case he wins. The sage told the king that he had a very modest claim and being a man of few material needs, all he wished was a few grains of rice. The amount of rice itself shall be determined using the chess-board in the following manner. One grain of rice shall be placed in the first square, two grains in the second square, four in the third square, eight in the fourth square and so on. Every square will have double the number of grains of its predecessor.

Upon hearing the demand, the king was unhappy since the sage requested only a few grains of rice instead of other riches from the kingdom which the king would've been happy to donate. He requested the sage to add other items to his prize but the sage declined.

So the game of chess started and needless to say the king lost the game. It was time to pay the sage his agreed-upon prize. As he started adding grains of rice to the chess board, the king soon realised the true nature of the sage's demands. By the 20th square, the number had reached one-million grains of rice and by the 40th square, it became one-trillion. The royal grainery soon ran out of grains of rice. The king realised that even if he provided all the rice in his kingdom and his adjacent kingdoms, he would never be able to fulfill the promised reward. The number of grains was increasing as a geometric progression and the total amount of rice required to fill a 64-squared chess board would be ...grains. This amount of rice would weigh about 461×10^9 tons.

Adapted from *Wikipedia.org*

Questions

1. Explain in your own words this legend.
2. "The number of grains was increasing as a geometric progression". What is a geometric progression ? Where do we use a geometric progression here ?
3. "By the 20th square, the number had reached one-million grains of rice." Calculate the exact amount.
4. Fill in the blank in the text : calculate the total amount of rice required to fill a 64-squared chess board.
5. (Série S) How many digits does it take to write this number ? (You may use the log function).
6. Knowing that 1,000 grains of rice weigh about 25g, show that the weight of the total amount of rice is about 461×10^9 tons. Compare it with the world production of rice which was 600 million tons in 2004.