Épreuve de section européenne

Classic fallacies[1] : a proof by induction

There are lots of "proofs" that claim to prove something that is obviously not true, like $1 + 1 = 1$ or $2 = 1$. These "proofs" contain some error that most people aren't likely to notice.

Example 1

$$\begin{aligned} -2 &= -2 \\ 4 - 6 &= 1 - 3 \\ 4 - 6 + \frac{9}{4} &= 1 - 3 + \frac{9}{4} \\ \left(2 - \frac{3}{2}\right)^2 &= \left(1 - \frac{3}{2}\right)^2 \\ 2 - \frac{3}{2} &= 1 - \frac{3}{2} \\ 2 &= 1 \end{aligned}$$

Example 2

This "proof" will attempt to show that all people in Canada are the same age, by showing by induction that the following statement (which we'll call "$S(n)$" for short) is true for all natural numbers n :

"In any group of n people, everyone in that group has the same age".

The conclusion follows from that statement by letting n be the number of people in Canada.

In any group that consists of just one person, everybody in the group has the same age, because after all there is only one person ! Therefore, statement $S(1)$ is true.

Let n be a natural number such that $S(n)$ is true. Let's prove that $S(n + 1)$ is true, that is to say "in any group of $n + 1$ people, everyone has the same age".

Let G be an arbitrary group of $n + 1$ people ; we just need to show that every member of G has the same age. To do this, we just need to show that, if m_1 and m_2 are any members of G, then they have the same age.

Consider everybody in G except m_1. These people form a group F of n people, so they must all have the same age (since we are assuming that, in any group of n people, everyone has the same age). Consider now everybody in G except m_2. Again, they form a group E of n people, so they must all have the same age.

Let m_3 be someone else in G other than m_1 or m_2. Since m_2 and m_3 each belong to the group F, they are the same age. Since m_1 and m_3 each belong to the group E, they are the same age. Since m_2 and m_3 are the same age, and m_1 and m_3 are the same age, it follows that m_1 and m_2 are the same age.

We have now seen that, if we consider any two people m_1 and m_2 in G, they have the same age. It follows that everyone in G has the same age.

The proof is now complete.

From various sources.

[1] A fallacy is an argument, or apparent argument, which professes to be decisive of the matter at issue, while in reality it is not.

Questions

1. Where does the mistake lie in the first example ?
2. Explain the method of proof by induction, with its different steps.
3. Find the mistake in the second example.
4. Do you know any examples of properties that you can prove by induction ?