Épreuve de section européenne

Rouse Ball's Fallacy

According to J. dePillis, George Pólya used to define Geometry as the science of correct reasoning on incorrect figures. But something is wrong with the diagram presented below. The reason I am so sure about that is because absolutely flawless[1] reasoning based on that figure leads to an absurd result. The question is : what is wrong ?

The construction is as follows. Form a right angle ADC and an obtuse angle DAE (away from DC) so that $DC = AE$. Since the two segments EC and AD are not parallel, their perpendicular bisectors are not parallel either. Let them meet in point O. Let K and H be the midpoints of EC and DA, respectively. Then

$$CO = EO \text{ as } \triangle CEO \text{ is isosceles}; \quad (1)$$
$$DO = AO \text{ as } \triangle ADO \text{ is isosceles}; \quad (2)$$
$$DC = AE \text{ by construction.} \quad (3)$$

By the SSS criterion, $\triangle OCD = \triangle OEA$. Therefore,

$$\measuredangle CDO = \measuredangle EAO. \quad (4)$$

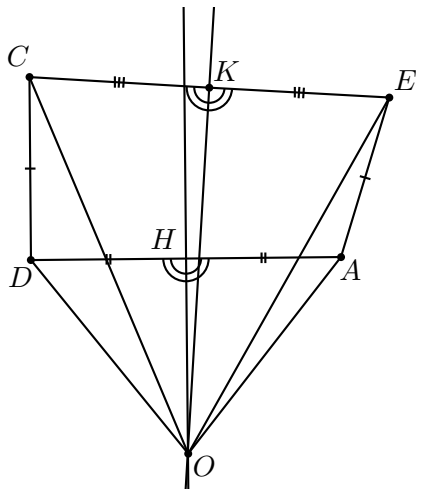
but also

$$\measuredangle ADO = \measuredangle DAO. \quad (5)$$

Subtracting (5) from (4) gives

$$\measuredangle ADC = \measuredangle DAE \text{ hence} \quad (6)$$
$$\measuredangle DAE = = 90^\circ. \quad (7)$$

We arrive at an absurd conclusion that the obtuse angle DAE is in fact right in contradiction with the construction.



Adapted from *http ://www.cut-the-knot.org/Curriculum/Fallacies*

Questions

1. Explain the reason why CEO and ADO are isosceles triangles.

2. What does the SSS criterion say ? What other criterions of the same kind do you know ?

3. Why can we say that $\measuredangle CDO = \measuredangle EAO$?

4. Draw the figure again. Can you see what is wrong ?

[1] with no mistake