Épreuve de section européenne

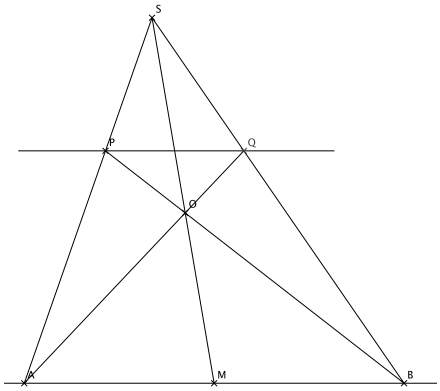
Steiner's parallels problem

In his famous book published in 1833 in Berlin, Jacob Steiner proved that all geometrical constructions that are done with straight-edge and compass can be done with straight-edge alone, provided that a fixed circle with its center is included in the picture. Here is one example of a construction using the straight-edge alone.

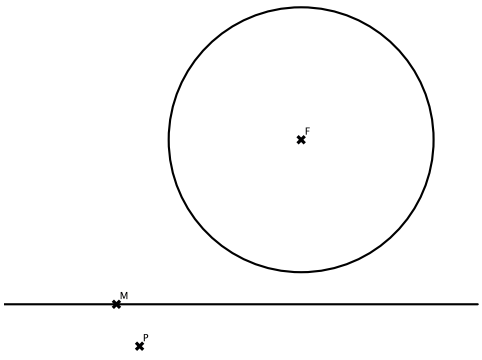
Problem : *To draw through a given point P the parallel to a given line, using a straight-edge alone.*

Steiner distinguished two cases : 1*a*. construction of the parallel to a directed straight line ; 1*b*. construction of the parallel to an arbitrary straight line.

1*a*. A directed straight line is understood to mean a straight line in which two points A and B and the midpoint M of the segment joining them are known. In order to draw the parallel to such a line through a given point P, we draw AP, choose a point S on the extension of AP, connect this point with B and M, draw BP, and draw the straight line AO through the point of intersection of BP and MS in such a manner that AO cuts BS at Q. PQ is then the desired parallel. [...]



1*b*. We connect a given point M of the given straight line g with the center F of the given fixed circle f and designate the points of intersection of the connecting line and f as U and V. The points F, U and V make the line FM a directed line. In accordance with 1a, we draw a parallel to FM in such a manner that it cuts f at X and Y and g at a point A. If we then draw the diameters XFX' and YFY' and connect the end points X' and Y', the connecting line intersects the given line at a point B in such a manner that $MA = MB$ and g, defined by the three points A, M and B is then a directed line. This makes it possible to determine the parallel to g in accordance with 1a.



Adapted from *100 Great problems of elementary mathematics* by Heinrich Dörrie, Dover, 1965

Questions

1. What is a directed straight line ?
2. Can you find two homotheties implied in construction 1a that map A on Q and P on B ?
3. The second figure is incomplete. Complete it according to the text.
4. Justify that FUV and AMB are directed lines.