Épreuve de section européenne

Harshad numbers

A Harshad number is an integer that is divisible by the sum of its digits. Harshad numbers were defined by D. R. Kaprekar, a mathematician from India. The word "Harshad" comes from the Sanskrit harsa (joy) + da (give), meaning joy-giver.

All 1-digit numbers are Harshad numbers.

The first 50 Harshad numbers with more than one digit in base 10 are : 10, 12, 18, 20, 21, 24, 27, 30, 36, 40, 42, 45, 48, 50, 54, 60, 63, 70, 72, 80, 81, 84, 90, 100, 102, 108, 110, 111, 112, 114, 117, 120, 126, 132, 133, 135, 140, 144, 150, 152, 153, 156, 162, 171, 180, 190, 192, 195, 198, 200.

For a prime number to also be a Harshad number, it must be a 1-digit number. Otherwise, the digits of the prime will add up to a number that is more than 1 but less than the prime, and obviously, it will not be divisible.

Although the sequence of factorials starts with Harshad numbers in base 10, not all factorials are Harshad numbers. 432! is the first that is not.

The integer 1729 is a Harshad number, but it's also an interesting number for another reason. It is the smallest number expressible as the sum of two cubes in two different ways.

Adapted from Wikipedia, the free encyclopedia, and various sources.

Questions

1. Check that 20, 150 and 198 are Harshad numbers.
2. Explain why all 1-digit numbers are Harshad numbers.
3. Explain the rule about prime numbers being Harshad numbers.
4. The n-th factorial is the product of all integers from 1 to n, that is $n! = 1 \times 2 \times 3 \times \ldots \times n$. Compute the first seven factorials and check that they are Harshad numbers.
5. Find two different ways to write the integer 1729 as the sum of two cubes.