Épreuve de section européenne

Completing the square

What must be added to $x^2 + 6x$ to make the result a perfect square ?

Fig. 129 represents a rectangle $(x+6)$ in. long, x in. high ; ∴ [therefore] its area $= x(x+6)$ sq. in. $= (x^2 + 6x)$ sq. in.

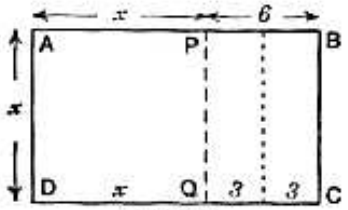

Fig. 129.

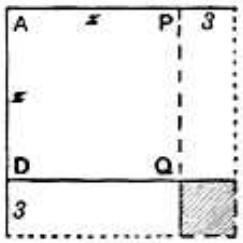

Fig. 130.

Transpose half the rectangle $PBCQ$ and fit it to DQ, as in Fig. 130. This gives a square of side x in., bordered with two rectangles, each of width $\frac{6}{2}$ in. $= 3$ in. To complete the square, add the shaded area in Fig. 130 ; this is a square of side 3 in., area 3^2 sq. in. $= 9$ sq. in. The result is a square of side $(x + 3)$ in., area $(x + 3)^2$ sq. in. $= (x^2 + 6x + 9)$ sq. in.

Hence, to $x^2 + 6x$, add $(\frac{6}{2})^2 = 9$; then the sum is $(x + 3)^2$.

To complete the square, start by looking for an expression whose square is of the required form.

Adapted from *School certificate Algebra*, by C.V. Durell, 1938.

Questions

1. What is the meaning of the expression "completing the square" ?
2. Explain the meaning of "in." and "sq. in.".
3. Use the two figures to explain the whole process in your own words.
4. Apply the same method to the expression $x^2 + 10x$.
5. Apply the method to the expression $x^2 + bx$, where b is any positive number.
6. "Completing the square" is an important step in the proof of the formula for the solutions of a quadratic equation.
   Give these formulas in the special case $x^2 + bx + c = 0$, where b and c are two real numbers.