Baccalauréat, série St — Session de juin 2011

Épreuve de section européenne

Convergency of series

An expression in which the successive terms are formed by some regular law is called a series ; if the series terminates at some assigned term it is called a finite series ; if the number of terms is unlimited, it is called an infinite series. [...]

If we can find the sum of the first n terms of a given series, we may ascertain whether it is convergent or divergent by examining whether the series remains finite, or becomes infinite, when n is made indefinitely great.

For example, the sum of the first n terms of the series

$$1 + x + x^2 + x^3 + \dots \text{ is } \frac{1 - x^n}{1 - x}.$$

If x is numerically less than 1, the sum approaches the finite limit $\frac{1}{1-x}$, and the series is therefore convergent.

If x is numerically greater than 1, the sum of the first n terms is $\frac{x^n - 1}{x - 1}$, and by taking n sufficiently great, this can be made greater than any finite quantity ; thus the series is divergent.

From *Higher Algebra*, by Hall and Knight, 1964.

Questions

1. What kind of sequence is 1, x, x^2, x^3,... ?
2. Explain the concept of a series.
3. What is the meaning of the words "convergent" and "divergent" for a series ?
4. What can you say about the series $1 + x + x^2 + x^3 + \dots$ when $x = 1$? Is it divergent or convergent ?
5. Let u be an arithmetic sequence with common difference d and first term $a_1 = 0$.
   (a) Give the first 4 terms of the sequence.
   (b) Give a formula using d and n for the finite series $a_1 + a_2 + \dots + a_n$.
   (c) Discuss the convergency of the series $a_1 + a_2 + a_3 + \dots$.
6. In the text, x is implicitly positive. What could you say about the series $1 + x + x^2 + x^3 + \dots$ when $x = -1$?