Baccalauréat, série S Session de juin 2011

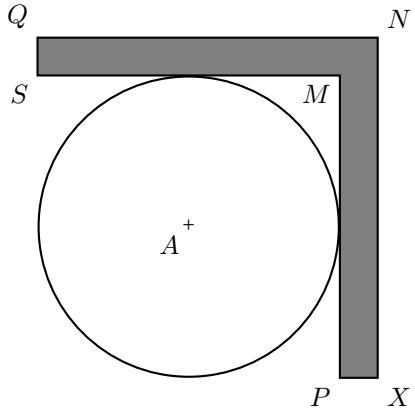
Épreuve de section européenne

Doubling the circle

The etymology of the word circle is from the Greek, kirkos, from the base ker- which means "to turn". The origin of the word "circus" is closely related as well.

The circle was known before the beginning of recorded history. Natural circles would have been observed, such as the Moon, the Sun, and a short plant stalk blowing in the wind on sand, which forms a circle shape in the sand. The circle is the basis for the wheel, which, with related inventions such as gears, makes much of modern civilization possible. In mathematics, the study of the circle has helped inspire the development of geometry, astronomy, and calculus. Early science, particularly geometry and astronomy, was connected to the divine for most medieval scholars, and many believed that there was something intrinsically "divine" or "perfect" that could be found in circles.

Let's consider a circle C_1 with centre A and radius r. How can we construct a circle C_2 with centre A and an area equal to the double of the area of C_1? One way to do this is to use a set-square. Place the set-square as in the picture on the right, with the two inner sides tangent to circle C_1. Then, rotate it around C_1. A pen placed at the right-angle of the set square (point M on the figure) will then trace the required circle C_2.



Adapted from Wikipedia, the free encyclopedia.

Questions

1. According to the text, quote a few natural circles.
2. If we draw a circle with radius the double of r, do we obtain the required circle C_2? Explain your answer.
3. Prove that we are looking for a circle of radius $\sqrt{2}r$.
4. Prove that the construction with the set-square gives the required circle C_2.
5. What should be the width $e = SQ = PX$ of the set-square so that a pen placed at corner N will draw a circle with area three times the area of C_1?