Épreuve de section européenne

Getting information from the equation of a curve

In most cases a great deal of information as to the shape and the limitations of the curve can be obtained by examining the equation. The following example illustrates this.

$$y = \frac{x^2}{1 + x^2}$$

In this case it is evident, since x^2 is always + for real values of x, that y is always +, and the curve is confined to the first two quadrants ; next that since x^2 is always +, $1 + x^2$ must be $> x^2$, and therefore y is always < 1. Hence the graph lies entirely in the strip between the axis of x and the parallel straight line $y = 1$.

The values of x corresponding to assigned values of y always occur in pairs equal in magnitude[1] and opposite in sign, since the equation only contains even powers of x; therefore the graph is symmetrical about the axis of y.

The curve goes through the origin, and in the neighbourhood of the origin y is much less than x; e.g. if $x = .1$, $y = .01/1.01 = 1/101$; therefore near the origin the graph keeps close to the axis of x. As x gets larger and larger, y gets nearer and nearer to 1, as is evident when the equation is written in the form $y = 1/(1 + 1/x^2)$; the term $1/x^2$ becomes less and less as x increases, and can be made as small as we please. Therefore y can be made as near to 1 as we please by increasing x sufficiently ; hence the line $y = 1$ is an asymptote to the curve.

Adapted from *Introduction to Infinitesimal Calculus*, G.W. Caunt, 1931

Questions

1. "Hence the graph lies entirely in the strip between the axis of x and the parallel straight line $y = 1$." Explain this sentence in your own words and illustrate it.

2. Explain why "the graph is symmetrical about the axis of y".

3. Prove that the equation can indeed be written in the form $y = 1/(1 + 1/x^2)$. Why does it show that "as x gets larger and larger, y gets nearer and nearer to 1" ?

4. Use the elements given in the text to draw a sketch of the curve.

5. Differentiate the function and deduce its variations. Are they consistent with the sketch from the previous question ?

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[1]magnitude=absolute value