Épreuve de section européenne

The Malthusian growth model

The Malthusian growth model, sometimes called the simple exponential growth model, is essentially exponential growth based on a constant rate of compound interest. The model is named after the Reverend Thomas Malthus, who authored *An Essay on the Principle of Population*, one of the earliest and most influential books on population.

Today, his model is expressed through the following form : $P(t) = P_0 e^{rt}$ where P_0 is the initial population, t is the time in years, and r is the growth rate, sometimes also called Malthusian parameter.

In 1798, Malthus posited[1] his mathematical model of population growth. His model, though simple, has become a basis for most future modeling of biological populations.

Malthus's observation was that, unchecked by environmental or social constraints, it appeared that human populations doubled every twenty-five years, regardless of the initial population size. Said another way, he posited that populations increased by a fixed proportion over a given period of time and that, with no constraints, this proportion was not affected by the size of the population.

Adapted from *Wikipedia* and an article by Steve McKelvey, Department of Mathematics, Saint Olaf College, Northfield, Minnesota

Questions

1. Using $P(t) = P_0 e^{rt}$ and the last paragraph of the text, show that, rounded to 4 d.p., $r \approx 0.0277$.
2. Compute the derivative of P, and write the differential equation verified by P.
3. The actual Malthusian model was that P_{n+1} is equal to $(1+r)P_n$ when P_n is the year n population, and r is equal to 0.0277. What sort of sequence is (P_n) ? Compare P_{25} and P_0. What do you notice ?
4. Malthus observed that the sequence representing the subsistence[2] for the population was increasing too, but in such a way that he could state the following table, for periods of 25 years :

Period of time	Index of population	Index of subsistence	n
1700-1725	100	100	0
1725-1750	200	200	1
1750-1775	400	300	2
1775-1800	800	400	3
1800-1825	1600	500	4

   Describe population and subsistence sequences. What consequence on the evolution could Malthus state from such a table ? What could he say about the solution to that problem ?
5. Considering that $P_{n+1} = (1 + r)P_n$ can be written as $P_{n+1} - P_n = rP_n$, can you explain the link between the models of questions 1 and 3 ?

[1] posited = stated as a principle
[2] feeding, especially