# Épreuve de section européenne

**Extreme value problems.**

Ferdinand is planning a bicycle tour through the Harz Mountains. He leaves his hotel in the morning and returns to his starting point. It should be clear that at the highest point of his trip, his bicycle is exactly horizontal. For if the front wheel were higher than the rear wheel, there must be a higher point ahead, and if it were lower, he must have come from a higher point.

It is just such an idea that underlies extreme value problems. If one has reached a maximum, then the steepness (or slope) of the curve must be zero. One could say that a zero slope is a *necessary* condition for the existence of an extreme value.

To use this knowledge for the actual calculation of extreme values, one requires formulas to express the slope of a curve. This was one of the strongest impulses in the development of modern mathematics namely the calculus invented independently by Leibniz and Newton.

Here is an example : where does the function $-x^2 + 6x + 10$ assume its greatest value ? Figure 1 shows that the function's graph first rises and then falls.

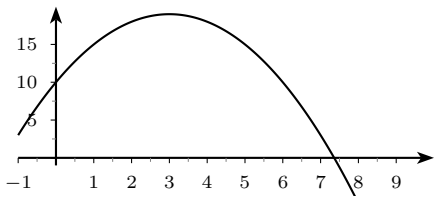


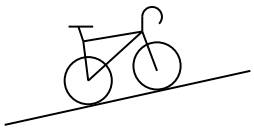

Figure 1: The function $-x^2 + 6x + 10$

But where does it reach its maximum ? Using the rules for the derivative, one knows that the slope at a point $x$ is given by $-2x + 6$. And this expression vanishes for $x = 3$. Therefore, the maximum value is achieved at $x = 3$. (One must be careful in solving such problems that one has not accidentally found a minimum of the function. After all, Ferdinand's bicycle may have been horizontal at the lowest point of his trip as well.)

Adapted from *Five-minutes mathematics* by E. Behrends.

## Questions

1. On the picture below; show where the front wheel and rear wheel are. Using the picture, explain what the slope is.



2. Explain in your own words (or with a picture) that the bicycle is horizontal at the highest point of Ferdinand's trip.
3. Draw a curve having zero slope at a point that is neither its maximum nor its minimum.
4. The altitude $h$ of Ferdinand's position is given as a function of the time $t$ elapsed from the start of his trip by :

$$h(t) = t^3 - 6t^2 + 9t + 2$$

(the altitude is in hundred metres and the time in hours, in the interval $[0; 5]$).

   (a) What is the altitude of the hotel ?
   (b) At what time will Ferdinand be back at the hotel ?
   (c) What was the highest point of his trip ? At what time did he reach it ?