## Épreuve de section européenne

Eratosthenes' measurement of the Earth

The most famous scientific achievement of Eratosthenes was his measurement of the earth. Eratosthenes improved a method that, according to Archimedes, some had tried, in order to calculate the circumference of the earth.

He observed that at Syene, at noon, at the summer solstice, the sun cast no shadow from an upright gnomon[1], while at the same moment the gnomon fixed upright at Alexandria (taken to be on the same meridian with Syene) cast a shadow corresponding to an angle between the gnomon and the sun rays of 1/50th of a complete circle or four right angles.

The sun rays are of course assumed to be parallel at the two places represented by S and A in the annexed figure. If α be the angle made at A by the sun rays with the gnomon (OA produced), the angle SOA is also equal to α, or 1/50th of four right angles.
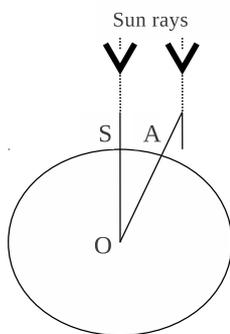
Now the distance from S to A was known by measurement to be 5,000 stades; it followed that the circumference of the earth was 250,000 stades. Theon of Smyrna reported that Eratosthenes corrected it to 252,000 stades for some reason, perhaps in order to get a figure divisible by 60 and, incidentally, a round number of stades per one degree. On the basis that the length of a stade was 157.5 meters, 252,000 stades work out to be 24,662 miles, and the diameter of the earth to be about 7,850 miles, only about 50 miles shorter than the true polar diameter, a surprisingly close approximation, however much it owes to happy accidents in calculation.

From *A History of Greek Mathematics*, by Sir Thomas Heath

Questions

1. Assuming that the Earth is a perfect sphere, and using Eratosthenes reasoning, prove that its circumference is, indeed, 250 000 stades.

2. Explain why $\angle SOA = \alpha$. Hence deduce the value of α in degrees.

3. By taking the value given by Theon of Smyrna, calculate the round number of stades per degree.

4. Use the last part of the text to calculate the number of kilometres in one mile, rounded to 3 d.p.

5. Name two mathematicians mentioned in the text. Do you know any other mathematicians or scientists from the Antiquity ? Can you mention some of their works ?

[1]Gnomon: A stick of wood that shows the time of the day by casting its shadow on a plane surface.