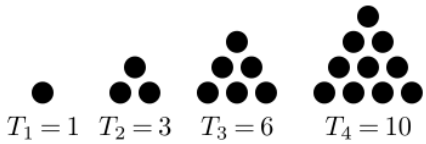
Épreuve de section européenne

Triangular numbers

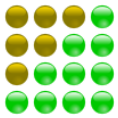
A triangular number gives the number of dots that can form an equilateral triangle, as in the diagram on the right. The n-th triangular number T_n is the number of dots in an equilateral triangle with n dots on one side. The sequence of triangular numbers is: 1, 3, 6, 10...



The triangular numbers are given by the following explicit formulas:

$$T_n = \sum_{k=1}^{n} k = 1 + 2 + 3 + \ldots + n, \text{ which is also equal to } \frac{n(n+1)}{2}.$$

Triangular numbers have a wide variety of properties:



- An integer x is triangular if and only if $8x + 1$ is a square.
- The sum of two consecutive triangular numbers is a square number, with the sum being the square of their difference.

Adapted from www.math.csusb.edu and various sources

Questions

1. Give the next four terms of the sequence of triangular numbers.
2. Are 57 and 66 triangular numbers ?
3. (a) Using T_n and n, give the mathematical formula translating the sentence : "The sum of two consecutive... of their difference".
   (b) Prove it.