Épreuve de section européenne

Divisibility criteria

Consider the algorithm below :

begin
  Input "Enter a positive integer number", n `// Example : n=354`
  m changes to n ;
  while $m > 70$ **do**
    t changes to the quotient in the division of m by 10 `// Example : t=35`
    u changes to the remainder in the division of m by 10 `// Example : u=4`
    m changes to $t - 2 \times u$;
  end
  if n *is in the multiplication table of* 7 **then**
    Output "The number n is a multiple of 7."
  else
    Output "The number n is not a multiple of 7."
  end
end

From various sources

Questions

1. (a) Try the algorithm with $n = 62755$, then with $n = 461258$. Fill in a table showing the different values taken by m, t and u.
   (b) Try the algorithm with a number that is not a multiple of 7.
2. Prove that n is a multiple of 7 if and only if $t - 2u$ is a multiple of 7. Hint : write $n = 10t + u$.
3. What other divisibility criteria do you know ?
4. A 3-digit number abc is a multiple of 11 if and only if $a + c \equiv b[11]$. Prove this property.