Épreuve de section européenne

Multiplicative Persistence

Take a whole number, and multiply its digits together. Repeat the operation with the answer, and repeat again until a single digit is reached. The number of steps required is called the multiplicative persistence of the number.

For all one-digit numbers, from 0 to 9, the multiplicative persistence is 0. The two-digit number 10 is the smallest number with multiplicative persistence 1. And for 268, the multiplicative persistence is 4, since $2 \times 6 \times 8 = 96$, then $9 \times 6 = 54$, then $5 \times 4 = 20$ and finally $2 \times 0 = 0$, which is a one-digit number. The order of the digits has no influence on the number of steps: 286 or 628 and 862 have the same multiplicative persistence as 268.

The smallest numbers with multiplicative persistence 1 to 10 are:

1	2	3	4	5	6	7	8	9	10
10	25	...	...	679	6 788	68 889	2 677 889	26 888 999	3 778 888 999

The smallest number with multiplicative persistence of 11 is 277 777 788 888 899. No number less than 10^{233} has a greater persistence and it is conjectured that there exists an upper limit to the multiplicative persistence of any number.

Adapted from *The Penguin Dictionary of Curious and Interesting Numbers*, David Wells, 1997

Questions

1. Compute the multiplicative persistence of the number 976.
2. What is the multiplicative persistence of a number:
   (a) containing the digit 0?
   (b) containing no digit 0, but simultaneously an even digit and the digit 5?
3. (a) Find the multiplicative persistence of each number from 26 to 40.
   (b) Deduce the smallest number with multiplicative persistence 3.
4. The smallest number with multiplicative persistence 4 lies between 70 and 80. Find this number.
5. In the second row of the table, all the numbers have their digits in increasing order. Explain why.