Baccalauréat, toutes séries Session de juin 2013

Épreuve de section européenne

The vesica piscis and the golden ratio

When two circles intersect, the convex region bounded by two circular arcs is called a *lens*. If the two circles have the same radius and each circle passes through the center of the other, the lens is called a *vesica piscis*. The vesica piscis arose in Euclid's *Elements* when constructing an equilateral triangle or bisecting a given line segment.

To compute the area of the figure, we can proceed as follows:

- Compute the area of the circular sector ABC centered at B;
- add the area of the circular sector BAC centered at A;
- subtract the area of the equilateral triangle ABC;
- double the result!

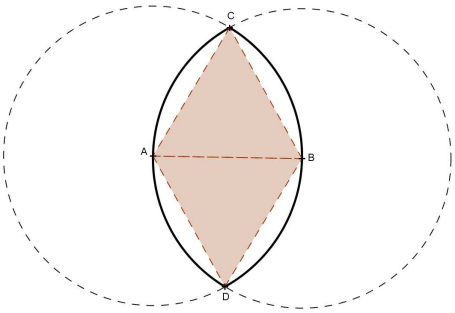

Figure 1

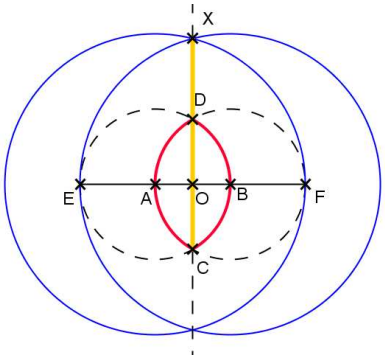

Figure 2

If we enclose the vesica piscis in a pair of intersecting circles whose centers coincide with the centers of the vesica piscis circles (see figure 2 above) and if we let $OA = OB = 1$ then we have another appearance of the golden ratio: $\frac{CX}{CD} = \frac{\sqrt{5}+1}{2} = \Phi$.

Adapted from *Icons of Mathematics* by Claudi Alsina, Roger B. Nelsen

Questions

1. Explain the link between an equilateral triangle and a vesica piscis.
2. (a) Explain each step of the computation of the area of a vesica piscis.
   (b) Give the formula of the area of a vesica piscis in terms of the common radius r.
   (c) What is the perimeter of a vesica piscis in terms of r?
3. Describe the construction of figure 2 in a few words.
4. (a) Prove that $OD = \sqrt{3}$ and $OX = \sqrt{15}$.
   (b) Deduce that $\frac{CX}{CD} = \frac{\sqrt{5}+1}{2} = \Phi$.