Épreuve de section européenne

Is 64 equal to 65 ?

Divide an 8×8 checkerboard as in figure 1 below. When the pieces are rearranged to make a rectangle, as in figure 2, there is an apparent gain in area of one square unit. Why so ? In fact, the points J, P, N and L on figure 2 are not collinear, even though they seem to be. There is a space along the diagonal, the parallelogram $JNLP$, but so elongated that it is not discernible.

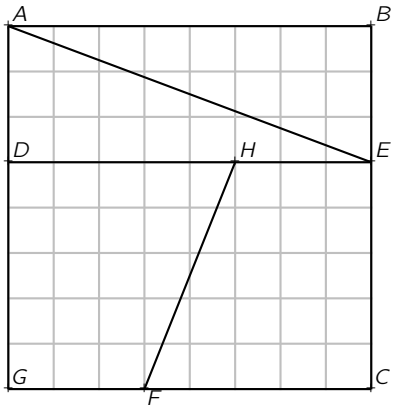

Figure 1

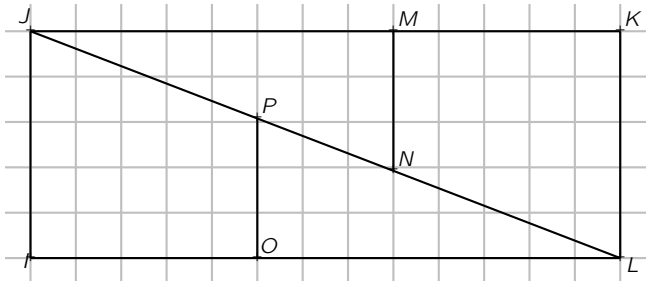

Figure 2

Sam Loyd Jr was the first to discover that the four pieces could be put together in a third way, so that the area is reduced to 63 squares.

Adapted from *Mathematics, magic and mystery* by Martin Gardner

Questions

1. Associate the four parts in the two pictures.
2. Compute the areas of the square and the rectangle and deduce the area of the parallelogram $JNLP$.
3. Explain the title of this document.
4. According to the text, what other number is "equal" to 64 ?
5. Why does the area of the parallelogram seem so counter-intuitive ?
6. Use coordinates to check that the points J, P, N and L on figure 2 are not collinear.