Épreuve de section européenne

Different sorts of proofs

A proof by counterexample is a way of showing that a given statement cannot be correct by showing an instance that contradicts a universal statement.

So, if you are trying to prove the statement “All cheesecakes are baked in Alaska.” and you did not know whether to prove it by contrapositive (1) or contradiction (2), all I would have to do is to bake a cheesecake right in front of you here in Texas and then you would know that your efforts had been in vain.

(1) : In logic, the contrapositive of a conditional statement of the form "if A then B" is formed by negating both terms and reversing the direction of inference. Thus, the contrapositive of the statement "if A, then B" is "if not B, then not A." A statement and its contrapositive are logically equivalent: if the statement is true, then its contrapositive is true, and vice versa.
(2) : In logic, proof by contradiction is a form of proof that establishes the truth or validity of a proposition by showing that the proposition's being false would imply a contradiction.

Adapted from *Wikipedia*

Questions

1. a) Express the statement “All cheesecakes are baked in Alaska” using the logic conditional items “if…, then…”, by completing the blanks in the following sentence :
   “… a cheesecake ……, then it has been …….”

   b) Express its contrapositive and check that it is equivalent to the initial statement.

2. Give another illustration of proof by counterexample.

3. According to you, is the following proposition “all the numbers of the form $P(n) = n^2 - n + 41$, where n is a natural number, are prime numbers” true or wrong?

4. Have you ever met other types of proofs ?