Épreuve de section européenne

Narcissistic numbers

In the recreational number theory, a narcissistic number is a number that is the sum of its own digits each raised to the power of the number of digits.

For instance, the number 371 is narcissistic: if you take the digits 3, 7 and 1, raise each to the third power, and add the powers together, you will get the original number: $3^3 + 7^3 + 1^3 = 27 + 343 + 1 = 371$

The digits of the number look in the mirror with operations, so to speak, and see only themselves.

In *A Mathematician's Apology*, G. H. Hardy wrote: "*There are just four numbers, after unity, which are the sums of the cubes of their digits:*

$153 = 1^3 + 5^3 + 3^3$

$370 = 3^3 + 7^3 + 0^3$

$371 = 3^3 + 7^3 + 1^3$

$407 = 4^3 + 0^3 + 7^3$

These are odd facts, very suitable for puzzle columns and likely to amuse amateurs, but there is nothing in them which appeals to the mathematician".

If for the number n with digits d_k, d_{k-1}, ..., d_1, we have: $n = {d_k}^m + {d_{k-1}}^m + ... + {d_1}^m$ for an integer m different from the number k, then n is not a narcissistic number, but is called a perfect digital invariant.

For example, the decimal number 4150 has four decimal digits and is the sum of the fifth powers of its decimal digits:

$4150 = 4^5 + 1^5 + 5^5 + 0^5$

so it is a perfect digital invariant but not a narcissistic number.

Adapted from Algebra II for dummies and Wikipedia.

Questions

1. Explain with your own words what a narcissistic number is.
2. Prove that 1634 is narcissistic. What about 8208 ?
3. Prove that 1-digit numbers are narcissistic.
4. Find the power that makes 4151 a perfect digital invariant.
5. Show that 18 is not a perfect digital invariant.