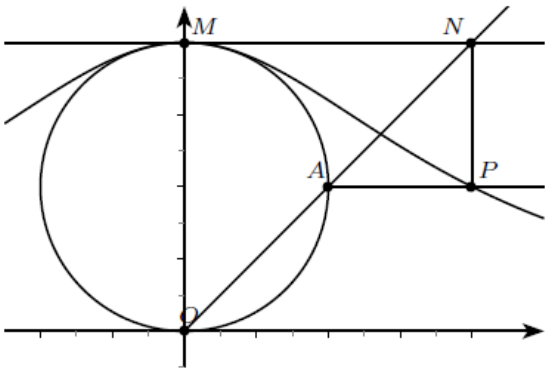
Épreuve de section européenne

The witch of Agnesi

Maria Agnesi was born in 1718 in Milan and died in 1799. She was the daughter of a wealthy silk merchant, Pietro Agnesi, and the eldest of his 21 children. Maria was recognized as a child prodigy very early; she could speak both Italian and French at five years of age. When she was fifteen, her father began to regularly gather in his house a circle of the most learned men in Bologna, before whom she read and maintained a series of theses on the most abstruse philosophical questions. Maria was very shy in nature and did not like these meetings. Her father agreed to let her live in an almost convent-like semi-retirement, avoiding all interactions with society and devoting herself entirely to the study of mathematics. During that time, Maria studied both differential and integral calculus.

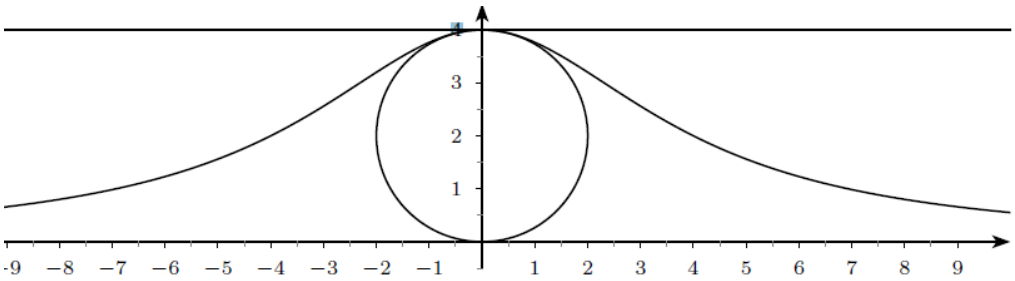
In 1748, she published her most famous work, *Instituzioni analitiche ad uso della gioventù italiana*. Among other things, she discussed in this book a curve earlier studied and constructed by Pierre de Fermat and Guido Grandi. Grandi called the curve *versoria* in Latin and suggested the term *versiera* in Italian. The name *"witch"* derives from a mistranslation of the term *averisera* ("versed sine curve" from the Latin *vertere*, "to turn") in the original work as *avversiera* ("witch" or "wife of the devil") in an 1801 translation of the work by John Colson.



The witch of Agnesi is the curve defined as follows: starting with a fixed circle, a point O on the circle is chosen; the point M is diametrically opposite to O. For any point A different from O on the circle, the secant line OA is drawn. The line OA intersects the tangent of M at the point N. The line parallel to OM through N and the line perpendicular to OM through A intersect at P. As the point N changes, the path of P is the witch.

The Cartesian equation of the witch of Agnesi is given, for a circle of radius a by : $y = \dfrac{8a^3}{x^2+4a^2}$

The curve $y = \dfrac{64}{x^2+16}$:



Idea from Professor Stewart's Cabinet of Mathematical Curiosities
Adapted from Wikipedia and The MacTutor History of Mathematics archive

Questions

1. How many brothers and sisters did Maria Agnesi have?
2. Explain why this curve is called a "witch".
3. Explain with your own words the construction of the curve, starting with a fixed circle.
4. Explain why the curve is asymptotic to the line tangent to the fixed circle through the point O.
5. What would the Cartesian equation of the curve constructed on a circle of diameter 1 be?
6. Compute the derivative of the function that gives the example above $y = \dfrac{64}{x^2+16}$ and give the corresponding table of variations.
7. Was it usual for a woman to study maths at that time ?