Épreuve de section européenne

Compound Interest Law

You will probably be familiar with two methods of payment of interest on borrowed or invested money, called simple and compound interest. In each of them the interest amounts to a fixed ratio to the magnitude of the sum of the money involved, called the principal. But while with simple interest the principal remains the same from year to year, with compound interest the interest is added to the principal at the end of each year and the interest for the succeeding year is calculated on the sum of the principal and interest.

Let P be the amount of money invested with compound interest and r the rate per cent per annum. The total at the end of the first year is $P_1 = P\left(1+\dfrac{r}{100}\right)$. And the total at the end of the second year is $P_2 = P\left(1+\dfrac{r}{100}\right)^2$, and so on until P_n at the end of the nth year (n being a natural number).

Suppose now that the compound interest is added at the end of each month, instead of at the end of each year. The rate for the year (called annual nominal rate) is still r, giving a rate for each month of $\dfrac{r}{12}$, so that the total at the end of the first year is $Q = P\left(1+\dfrac{r}{12\times100}\right)^{12}$.

Adapted from *"Calculus"*, Abott and Neill, Teach Yourself, 2003

Questions

1. Suppose you invest £1000 at the rate $r = 3$ per cent per annum.
   a) Compute the interest you will get at the end of the first year, and at the end of the second year with simple interest.
   b) Answer the same question with compound interest.

2. Give the formula for P_n. What kind of sequence is $\left(P_n\right)_{n\in\mathbb{Z}}$?

3. You invest a capital at the rate $r = 3$ per cent per annum with compound interest. How many years must you wait until your capital is doubled?

4. Now you invest a capital at the rate r per cent per annum with compound interest, and you want your capital to be doubled after 5 years. Compute the value of r, rounded to 2 decimal position.

5. You invest £1000 with compound interest added at the end of each month, with an annual nominal interest $r = 3$ per cent.
   a) Compute your capital after 1 year.
   b) Calculate the percentage of increase (called the annual effective interest) of the capital for this year, and compare it to the annual nominal interest.