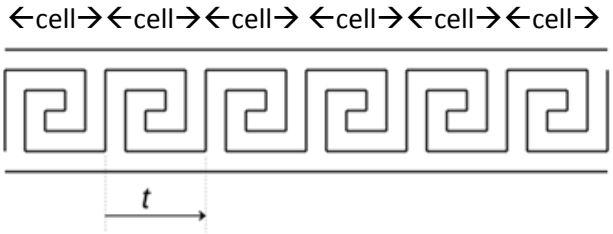
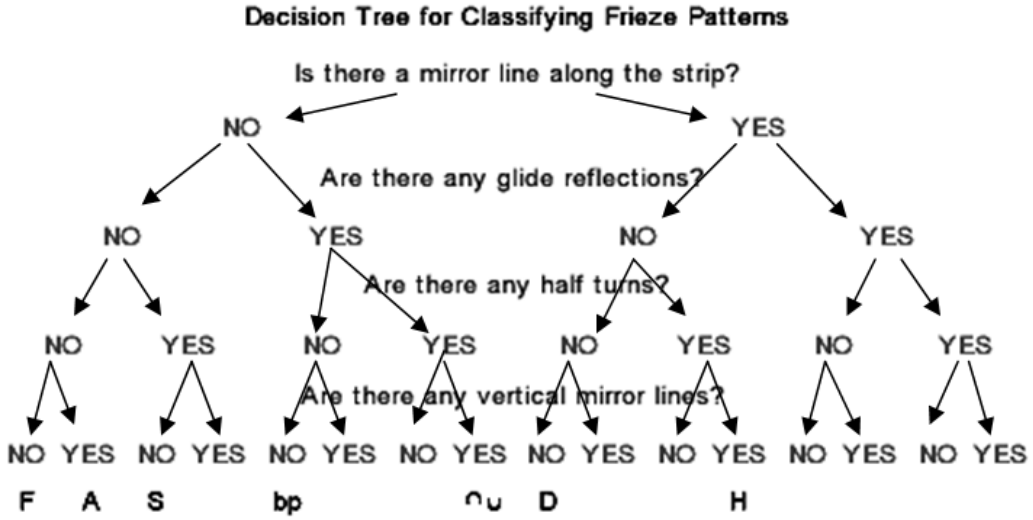
Épreuve de section européenne

Friezes

Repeating strip patterns, called friezes, occur all over the world in border decoration in buildings, textiles etc. All frieze patterns have a section of the pattern which is repeated alongside itself (we call this a translation). In order to distinguish one frieze pattern from another one we first need to find the smallest translation length in the strip. This defines the 'cell' which is the smallest piece of the pattern to be repeated by translations. Perhaps surprisingly, mathematicians say that there are only seven different frieze patterns.



Apart from translation, there are four other symmetries which transform the strip into itself. We shall call the four symmetries H, V, R and G, namely H for the reflection in a mirror line along the strip (horizontal reflection, a symmetry which occurs in the letter D); V for the reflection in a mirror line perpendicular to the strip (vertical reflection, as in A); R for the rotation by a half turn (as in the letter S); and G for a glide reflection (as in bp). Frieze patterns are classified according to whether each of the four symmetries do or do not occur, and you can use a decision tree to do this classification.



In the last line are written the letters which can be a cell for the given frieze. A blank denotes impossibility.

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Questions

1. Draw three friezes: the first one admitting only a vertical mirror line, the second one admitting only a horizontal mirror line, and the last one admitting only a half turn.
2. A glide reflection is a reflection followed by a translation parallel to the axis of this reflection. Use the example given in the text ("as in bp") to find the image of the letter “d” by a glide reflection.
3. Where would you place the letter K on the last line? Explain why.
4. Imagine and draw a frieze, other than the one made with "bp", that admits a glide reflection.
5. According to the tree, it is impossible to have a frieze with a mirror line along the strip, a vertical mirror line, and no half turn. Explain why.