Épreuve de section européenne

How many candies do you have?

First prepare

You can perform this trick with any number of any kind of small items, say candies in a bag. You should have a sufficient supply of them. Besides this, you – the mathemagician – also need two volunteers for the trick; we shall call them Alexander and Barbara. If you are not sure that your volunteers are good in mental multiplication of small numbers, have a pen and paper or a calculator ready for them.

First of all, you secretly choose two numbers (positive integers), add 1 to the first of them and multiply this number with the second and remember the result. For example, if you have chosen the numbers 6 and 2, you calculate $(6+1) \times 2 = 14$. Remember 14 and you are prepared to do the trick!

Now it's time to do the trick...

First you instruct Alexander to take any number of candies from the bag. You must turn away so that at no moment you see how many candies the two volunteers have. Now, Alexander shows Barbara his candies and she counts them and then takes 6 times as many from the bag. For example, if Alexander took 9 candies, she takes 54.

Now, tell Alexander to give Barbara two of his candies (in the described example this leaves Alexander with 7 and Barbara with 56 candies). He may be grumbling now because Barbara has so many candies, but the next step will make Barbara grumble: tell her to give to Alexander from her candies 6 times as many as he still has. In our example, this means Barbara giving $6 \times 7 = 42$ candies to Alexander.

Finally, it's your turn to make a dramatical announcement that you know how many candies Barbara still has – the number is 14 as computed at the beginning.

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Questions

1. Give the number of candies Barbara ends up with if you choose the numbers 9 and 4.

2. Let us suppose that you choose the numbers 9 and 4, and that Alexander takes 5 candies from the bag. Complete this table:

	1st step	2nd step	3rd step	4th step
number of candies Alexander has	5			
number of candies Barbara has				

3. Still with the numbers 9 and 4 chosen at the beginning, we suppose now that Alexander takes n (n is a positive integer). Complete this table:

	1st step	2nd step	3rd step	4th step
number of candies Alexander has	n			
number of candies Barbara has				

4. Now you choose the numbers m and s (positive integers) in the beginning. How many candies should you have in the bag before starting if you forbid Alexander taking more than 10 candies?