Baccalauréat, Série S et ES Session de juin 2013

Épreuve de section européenne

Exploring Bayes' s theorem

In essence, Bayes's theorem describes the relationship between two conditional probabilities: $P_B(A)$ and $P_A(B)$. In other words, the theorem allows the events in a conditional probability to be "switched" - that is, it allows us to find the probability of A given B when the probability of given A is known. Unfortunately, the distinction between $P_B(A)$ and $P_A(B)$ is easy to miss, even by professional physicians and medical school staff. Consider the Mammogram problem (Gigerenzer and Hoffrage 195, adapted from Eddy 1982):

"The probability for breast cancer is 1% for a woman of age forty who participates in routine screening. If a woman has breast cancer, the probability is 80% that she will get a positive [result from a mammogram]. If a woman doesn't have breast cancer, the probability is 9.6% that she will also get a positive. A woman in this age group had a positive in a routine screening. What is the probability that she actually has breast cancer?"

Adapted from NCTM

Questions

1. Perspective 1: Expected frequencies over a group of 1000 patients.

   a. Fill in the following table with the expected frequencies rounded to the nearest integer:

	Positive test	Negative test	Total
Cancer			10
No cancer	95		990
Total			1000

   b. Give the answer to the problem.

2. Perspective 2: Probabilities
   Suppose that A represents the event that a woman participating in routine screening has breast cancer and that B represents a positive result for a mammogram.

   a. According to the text, give $P(A)$, $P_A(B)$ and $P_{\bar{A}}(B)$.
   b. Calculate $P(B)$ and deduce the answer to the problem.

3. Eddy (1982) reports that 95% of physicians who were asked this question estimated the correct answer to be between 70% and 80% (cited by Gigenrenzer and Hoffrage 1995).
   How can you account for the physicians' mistake in estimating the answer to the problem?