Épreuve de section européenne

Al-Khwarizmi's routine for quadratic equations

Imagine you have a single piece of carpet of unknown width, and of 10 units length. Suppose if you cut a strip of area 21 square units from the carpet, you will be left with a square. It is then possible to figure out the width of the carpet.

You begin by making a drawing of the carpet showing the various sizes. The unknown quantity is the width. Al-Khwarizmi calls this the 'root'. When we square it, that is, multiply it by itself, the answer will be an area. This is the unknown raised to the second power and is called the 'square'.

Using symbols, not words, the equation can be written as follows

$$W^2 + 21 = 10W.$$

To solve it, the routine is:

- Divide the numbers of roots (10) by two . . . . . answer 5.
- Multiply the answer (5) by itself . . . . . answer 25.
- Subtract the number, $25 - 21 = 4$ . . . . . answer 4.
- Take the square root of the latter . . . . . answer 2.
- Subtract from half the number of roots $(5 - 2)$ . . . . . answer 3.
- 3 is one of the roots we seek; the square is 9.

Adapted from *Number*, John McLeish, 1991

Questions

1. Illustrate the situation with a drawing.
2. Explain what is called "the number of roots" and "the number" in the routine.
3. (a) Apply the routine to find the roots of the equation $W^2 + 3 = 4W$.
   (b) Solve the same equation by the modern method. Do you find the same answers ?
4. Apply the routine to find the roots of the equation $W^2 + m = pW$, where m and p are two positive real numbers. Compare the result to the modern formula.
5. Add some instructions to the routine to find the second root.