Épreuve de section européenne

Binary subtraction and the NOT operator

Just as a decimal number is decomposed as powers of ten (for example $247 = 2 \times 10^2 + 4 \times 10^1 + 7 \times 10^0$), a binary number is decomposed as powers of two. For example, the binary number 1101 denotes the quantity

$$1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 8 + 4 + 1 = 13.$$

Computers carry out operations exclusively on binary numbers. For example, in binary $0+0 = 0$, $1 + 0 = 0 + 1 = 1$ and $1 + 1 = 10$. This last operation will be useful later on in this text.

A binary number made of 8 digits (all 0s or 1s) is called a *byte*.

The computer performs the subtraction of one number from another by changing the number subtracted into its complement and adding. In binary, the complement of a byte is given by the operator NOT. (There are occasional problems with the generation of extra 'carry' numbers at the end, but simple refinements cure them.) For example, if the computer is asked to calculate $243 - 95$, it will proceed as follows (note that 243 and 95 are respectively 11110011 and 01011111 in binary) :

$$11110011 - 01011111$$

is the same as

$$11110011 + \mathrm{NOT}(01011111)$$

which is

$$11110011 + 10100000$$

that is

$$110010011$$

.

The extra 1 at the end (the first digit on the left), which is the carry of the last $1 + 1$ addition, is moved round and added at the beginning (one of the refinements mentioned above), making the number we are seeking

$$10010100$$

which is indeed equal to $148 = 243 - 95$.

Adapted from *Number*, John McLeish, 1991

Questions

1. Check that the binary notations of 243 and 95 are respectively 11110011 and 01011111.

2. What is a *byte* ? What is the greatest decimal number that can be written with a byte ?

3. The example $\mathrm{NOT}(01011111) = 10100000$ shows that the complement of a byte can easily be obtained graphically. Deduce from this the binary value of $\mathrm{NOT}(00101001)$.

4. Give the decimal values of the binary numbers 00101001 and $\mathrm{NOT}(00101001)$. How does this result explain the name 'complement' for the operator NOT ?

5. Use the showcased method in the text to carry out the subtraction $52 - 41$.