## Épreuve de section européenne

Gambling on Horse Races

Dr Z recently developed various ways of making money by gambling on horse races. You bet on a horse, and if it wins you fairly share the total amount of money placed on the race with all the other lucky punters[1] that picked the right horse. Dr Z realized that, although people were reasonably good at picking out a horse that might be a winner, they were not good at working out the odds that the horse might finish in one of the top places. So, he came up with a system that calculated the chances of a horse finishing in one of these places. Supposing that enough people are betting, the Law of Large Numbers makes him consider it a reasonable assumption that, if $600 had been placed on *'Likely Lad'*, out of a total pool of $10,000, the chances of this horse winning were 600/10,000. He then used the rules of theoretical probability to work out the chances of the horse finishing in the top places.

Imagine there is a three-horse race involving *'Artful Dodger'*, *'Likely Lad'* and *'What a Palaver'*, and that the probabilities for each of them winning are 5/10, 3/10 and 2/10 respectively. The problem comes when working out what the chances of a particular horse coming second are, if another horse comes first. From the initial probabilities, you know that *'Likely Lad'* wins a race three times for every two times that *'What a Palaver'* wins. So you consider this situation to be just another race – although it is, in fact, a race for second place. With the information above, if *'Likely Lad'* and *'What a Palaver'* raced for second place five times, you would expect *'Likely Lad'* to perform the better in three of those races, and *'What a Palaver'* to perform the better in two of them. Therefore, if *'Artful Dodger'* comes first, the chances of *'Likely Lad'* coming second are 3/5 and the chances of *'What a Palaver'* coming second are 2/5. See the tree diagram below.
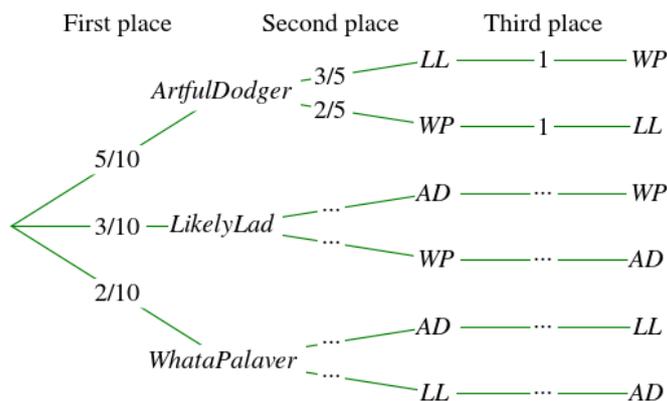
Similar arguments can be constructed to find the relevant probabilities if *'What a Palaver'* or *'Likely Lad'* comes first. Since this is a three-horse race, if *'Artful Dodger'* comes first, and *'Likely Lad'* comes second, then *'What a Palaver'* is certain to come third. Now, finally, you can help Dr Z with his calculations.

punters[1]: gamblers

Questions

1. If *'Likely Lad'* comes first, follow Dr Z's reasoning to show that the probability of *'Artful Dodger'* coming second is equal to 5/7.

2. Complete the above tree diagram.

3. a. Check then that the chances of *'What a Palaver'* coming in the first two places are 17/35.

   b. Suppose that $2,800 have been placed in bets on horses finishing in the top two, and $1,000 from this sum have been placed on *'What a Palaver'*. Should you rush to bet money on this horse?