Épreuve de section européenne

The Reaper's Problem
How wide should the strip be?

Mechanics and laborers who have no great skills in mathematics will often solve, in a practical way, some very difficult problems. I call the attention of our puzzlists to the clever way in which a couple of farmer adjusted their affairs.

A Texas ranchman, who owned more land than he could conveniently farm, leased half of a certain field to a neighbor.

This field was 2 miles long by 1 mile wide, but because of certain bad streaks which ran through the land it was decided that a fairer division would be obtained by cutting a band around the field than by dividing it in half.

I presume our puzzlists will find no great difficulty in determining the width of a border strip, to be cut all around that field, that will contain exactly half of the total crop.

There is a simple rule which will apply to any rectangular field: they said: "One quarter the difference between a short cut cross lots, and round by the road". Mathematicians will understand it better if we say: from the sum of the two sides subtract the diagonal of the field and divide the result by four.

From *More mathematical puzzles* by Sam Loyd, edited by Martin Gardner

Questions

1. Use the simple rule written in the last sentence to compute the width of the border strip, rounded to 3 d.p.

2. Using the former answer, show that the border contains approximately half of the total crop.

3. Let's denote by x the width of the border strip. Show that x is a solution of the quadratic equation
$$4x^2 - 6x + 1 = 0.$$

4. Use this equation to prove that the "simple rule" actually gives the correct answer.