Épreuve de Section Européenne

Pythagorean Triples

A Pythagorean triple consists of three positive integers *a*, *b*, and *c*, such that $a^2 + b^2 = c^2$. Such a triple is commonly written (*a*, *b*, *c*). A well-known example is (3, 4, 5). A right-angled triangle whose sides form a Pythagorean triple is called a Pythagorean triangle.

The general construction of Pythagorean triples has been known for a long time. Proclus, in his commentary to the 47th Proposition of the first book of *Euclid's Elements*, describes it as follows:

Certain methods for the discovery of triangles of this kind are handed down, one referring to Plato, and another to Pythagoras.

Plato's method starts with an even number:

- Take any even number (for example, let's start with 12) and make it one of the sides about the right angle;
- Bisect this number (so it's equal to 6), square it (36), then subtract unity (35) to form the other side of the triangle;
- Finally add two units to the last result to form the hypotenuse (37).

So the Pythagorean triangle has for dimensions (12 ; 35 ; 37).

Pythagoras' method starts with an odd number :

- make that odd number the smallest side about the right angle;
- then take its square, subtract unity and consider half that difference as the greatest side about the right angle;
- lastly add unity to this and so form the remaining side, the hypotenuse.

Adapted from a Wikipedia article

Questions

1. a. According to the text, who wrote the comment on the book *Euclid's Elements* ?
   b. What are even and odd numbers ?
   c. Is the triple $(1; 1; \sqrt{2})$ a Pythagorean triple ?
2. a. Using a method described in the text, find out the dimensions of a Pythagorean triangle whose smallest side is 6.
   b. Using a method described in the text, find out the dimensions of a Pythagorean triangle whose smallest side is 5.
3. a. Let (*a*, *b*, *c*) be a Pythagorean triple with a an even number. According to Plato's method, what are the values of b and c in function of a ?
   b. Check that the triple defined with these formulas is indeed a Pythagorean triple.