Épreuve de section européenne

The two sorcerers' towers

In a country far away from England, two sorcerers of a tribe have a discussion about a tower they want to build with stones. The heights of the stones are different, but the height of the first stone at the base is equal to 1 meter.

The first sorcerer says that if he piles[1] stones whose height is the reciprocal of the number of piled stones ($\frac{1}{2}$ for the second one, $\frac{1}{3}$ for the third one, $\frac{1}{4}$ for the fourth one…), his tower will reach the sky. The second sorcerer replies that if he piles stones whose height is the half of the last piled stone ($\frac{1}{2}$ for the second one, $\frac{1}{4}$ for the third one, $\frac{1}{8}$ for the fourth one…), his tower will also reach the sky.

To avoid a war between the two sorcerers, the chief of the tribe decides to call a mathematician for help.

After a few seconds, the mathematician explains that the first sorcerer is right, because it's possible to write first: $1+\frac{1}{2}+(\frac{1}{3}+\frac{1}{4})\geq 1+\frac{1}{2}+(\frac{1}{4}+\frac{1}{4})=1+\frac{1}{2}+\frac{1}{2}$ then, secondly : $1+\frac{1}{2}+(\frac{1}{3}+\frac{1}{4})+(\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8})\geq 1+\frac{1}{2}+(\frac{1}{4}+\frac{1}{4})+(\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8})=1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$. Then is continues with an infinity of $\frac{1}{2}$…

After that, the mathematician explains to the second sorcerer that the height of his tower is equal to the sum $1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+...+\frac{1}{2^{n-1}}=1\times\frac{1-(\frac{1}{2})^n}{1-\frac{1}{2}}$, with *n* tending to the infinite. This sum is easy to estimate.

From various sources

[[1] *empile* [2] *lance*]

Questions

a) What is "the reciprocal of an integer *n*"?
b) Explain the sentence "his tower will reach the sky".
c) The mathematician gave the calculations for the first two rows of the first tower. Give the next one and explain his proof with your own words.
d) Explain the calculation for the second tower, give the result and explain why the second tower had to disappoint the second sorcerer.
e) Do you know some scientists or mathematicians who lied to save their life ?