Épreuve de section européenne

The ABC Conjecture

The ABC Conjecture probes deep into the darkness, reaching at the foundations of math itself. First proposed by mathematicians David Masser and Joseph Oesterle in the 1980s, it makes an observation about a fundamental relationship between addition and multiplication. Yet despite its deep implications, the ABC Conjecture is famous because, on the surface, it seems rather simple.

It starts with an easy equation: $a + b = c$.

The variables a, b, and c, which give the conjecture its name, have some restrictions. They need to be whole numbers, and a and b cannot share any common factors, that is, they cannot be divisible by the same prime number. A prime number is a whole number that is only divisible by 1 and itself. The first prime numbers are 2, 3, 5, 7, 11, 13.

So, for example, if a is 6, which equals 2×3, then b could not be any number that is a multiple of 2 or 3. In this case, b could be 35, for example, which is 5×7. Now a and b do not share any factors, and we get the equation $6 + 35 = 41$.

The ABC Conjecture essentially says that when there are lots of prime factors on the left hand of the equation then, usually, there will be not very many on the right side of the equation. Here, there are 4 prime factors on the left-hand side, but only one on the right-hand side.

From *The Paradox of the Proof*, by Carolyn Chen

Questions

1. What is a prime number ? Give an example greater than 15.
2. Consider the sum 10+21.
   a) What are the prime factors of 10 and 21?
   b) Does this sum satisfy the conditions of the conjecture?
   c) Is the conjecture verified by this sum?
3. Could you come up with another example of a sum that verifies the conjecture?
4. What do you think of the sum 64+81=145.
5. Discuss the meaning of "lots of prime factors" and "not very many prime factors".