Épreuve de section européenne

A Pythagorean tragedy

There are tragedies in mathematics also, and one of these struck the very group of mathematicians who deserved a better fate.

The Pythagoreans had constructed, at least to their own satisfaction, a philosophy that asserted that all natural phenomena and all social and ethical concepts were in essence just whole numbers or relationships between whole numbers. But one day it occurred to a member of the group to examine the seemingly simplest case of the Pythagorean theorem.

Suppose each arm of a right triangle is 1 unit in length; how long, he asked, is the hypotenuse? The Pythagorean theorem says that the square of (the length of) the hypotenuse equals the sum of the squares of the arms. Hence, if we call *c* the unknown length of the hypotenuse, the theorem says that

$$c^2 = 1^2 + 1^2$$

or

$$c^2 = 2\,.$$

Now 2 is not a square number, that is, a perfect square, and so *c* is not a whole number. But it certainly seemed reasonable to this Pythagorean that *c* should be a fraction; that is, there should be a fraction whose square is 2. Even the simple fraction $\frac{7}{5}$ comes close to being the correct value because $\left(\frac{7}{5}\right)^2 = \frac{49}{25}$ and this is almost 2. However, simple trial does not easily yield a fraction whose square is 2. Then this Pythagorean became worried, and he decided to investigate the question of whether there is a fraction whose square is 2.

From *Mathematics for the Nonmathematician*, by Morris Kline

Questions

1. Explain the difficulty experienced by the Pythagorean mentioned in this text.
2. Give the definitions of the following words :
   a) right triangle ;
   b) arm of a triangle ;
   c) hypotenuse.
3. Draw the specific triangle mentioned in the document. What is the modern notation for the length of its hypotenuse?
4. Explain the sentence “the simple fraction $\frac{7}{5}$ comes close to being the correct value”.
5. Find an integer *a* such that the fraction $\frac{a}{7}$ is even closer to the correct value.