Episode 01 – Finding Pick's formula

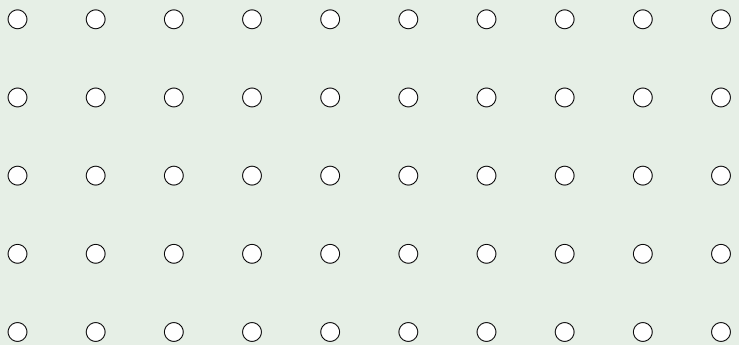
European section – Season 3

What is a lattice ?

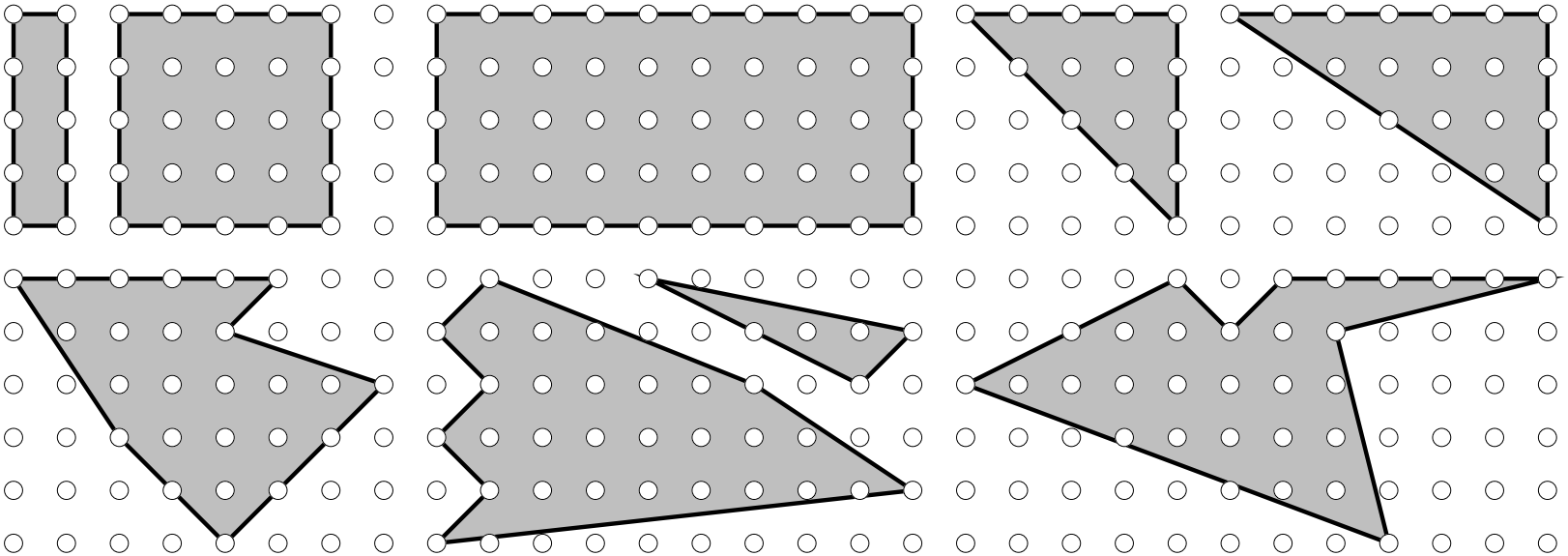
Definition (Lattice)

A lattice is a square grid, a discrete surface with a regular gap between any two points. A simple example is the set of the points with whole coordinates on a coordinate graph.

Example



Lattices polygons



Pick's formula

Definition (Notations)

For any polygon P in a lattice, we note B_P the number of points on its border and I_P the number of points strictly inside the polygon. The area of the polygon will be noted $\mathscr{A}_P$.

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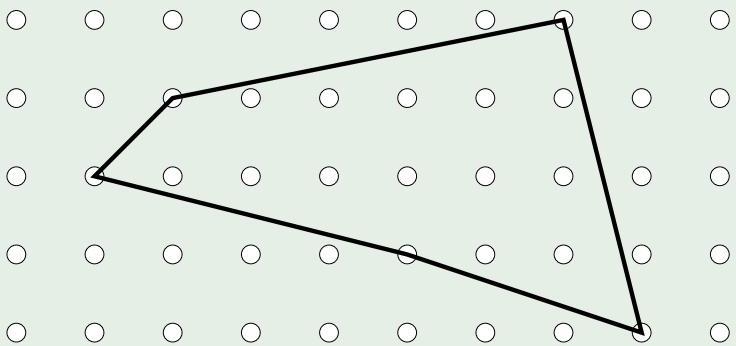

$$B_P =$$
$$I_P =$$

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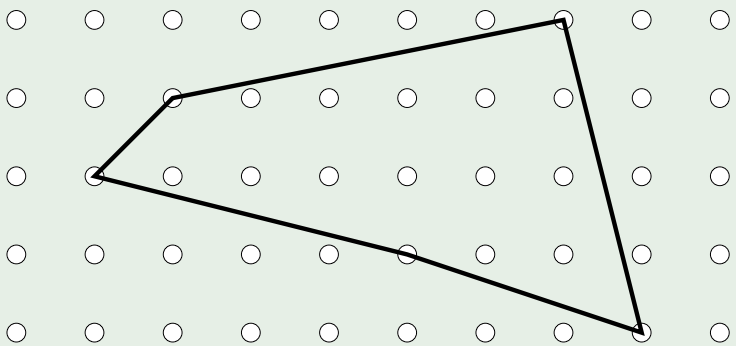

$$B_P = 5$$
$$I_P =$$

Pick's formula

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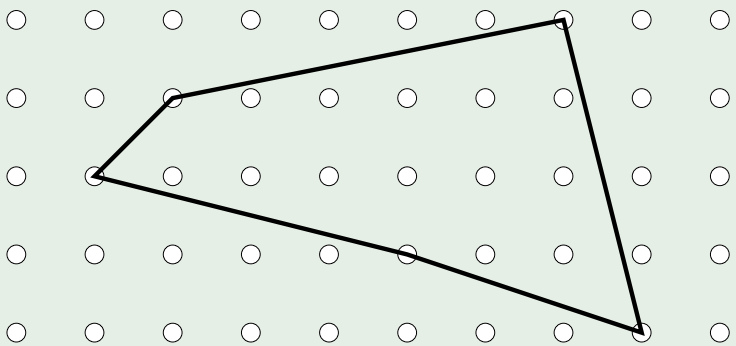

$$\begin{aligned} B_P &= 5 \\ I_P &= 13 \end{aligned}$$

Find a formula involving
the numbers B_P and I_P
to compute the area
of any lattice polygon.

Theorem

For any convex polygon P in a square two-dimensional lattice,

$$\mathscr{A}_P = \frac{1}{2}B_P + I_P - 1.$$

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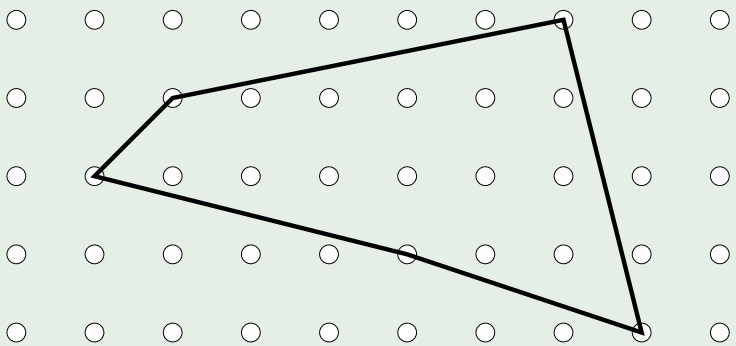

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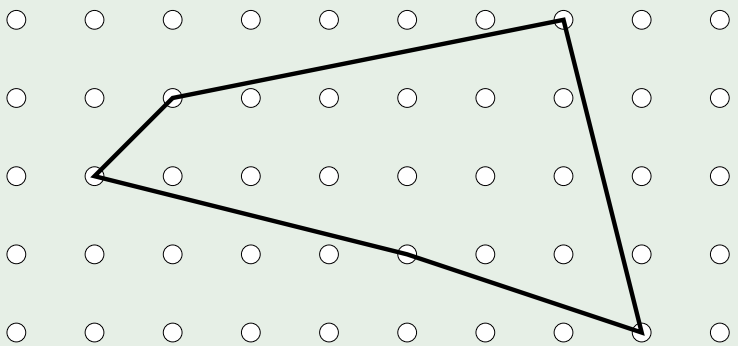

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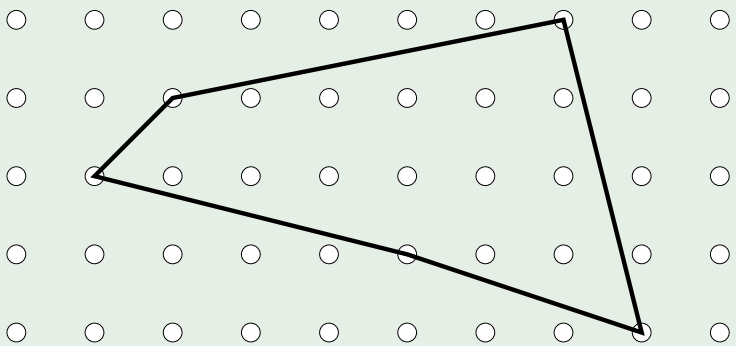

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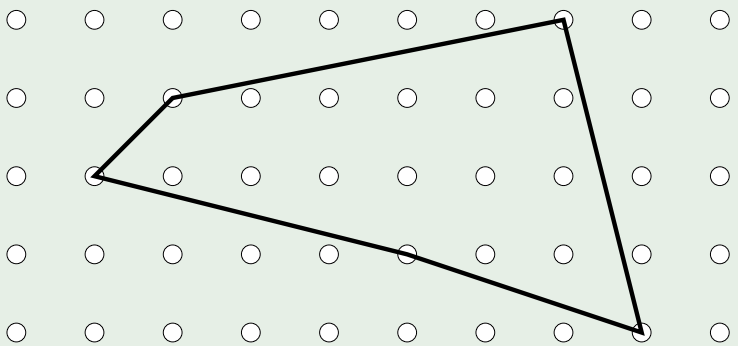

$$\begin{aligned} B_P &= 5 \\ I_P &= 13 \\ \mathscr{A}_P &= \frac{1}{2}B_P + I_P - 1 \\ \mathscr{A}_P &= \frac{5}{2} + 13 - 1 \\ \mathscr{A}_P &= \frac{29}{2} \end{aligned}$$